NBSIR 75-712

Solar Heating and Cooling in Buildings: Methods of Economic Evaluation

Rosalie T. Ruegg

Building Economics Section
Center for Building Technology
Institute for Applied Technology
National Bureau of Standards
Washington, D. C. 20234

July 1975

Final Report

U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

NBSIR 75-712

SOLAR HEATING AND COOLING IN BUILDINGS: METHODS OF ECONOMIC EVALUATION

Rosalie T. Ruegg

Building Economics Section
Center for Building Technology
Institute for Applied Technology
National Bureau of Standards
Washington, D. C. 20234

July 1975

Final Report

U.S. DEPARTMENT OF COMMERCE, Rogers C.B. Morton, Secretary
NATIONAL BUREAU OF STANDARDS, Richard W. Roberts, Director

PREFACE

This research was begun as a supporting economics effort for The Office of Housing Technology in the Center for Building Technology, Institute for Applied Technology, National Bureau of Standards. This report, prepared by the Building Economics Section, was written to summarize the findings of that research and to provide a background document that researchers and analysts can use for economic evaluations of solar heating and cooling systems in buildings. Comments on this presentation are invited. Input from the reader will be useful in establishing needs and objectives of future publications on the topic of life-cycle costs of space conditioning equipment.

Appreciation is extended to the solar energy consultants and those members of the CBT staff who reviewed the paper. Special appreciation is extended to Dr. Harold E. Marshall, Building Economics Section, for his valuable assistance throughout preparation of the paper.

ABSTRACT

This report addresses economic issues important to the design, acquisition, and evaluation of the costs to consumers, of solar heating and cooling systems in buildings. It explains and illustrates with simple, but realistic examples the use of life-cycle cost analysis and benefit-cost analysis to evaluate and compare the economic efficiency of solar and conventional energy systems. It also explains the conditions for making cost-effective tradeoffs in solar system/building design. By presenting the basic methods and assessing the appropriateness of alternative assumptions, the paper provides a resource document for researchers and analysts.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

1.1 Background

The widespread use of solar heating and cooling systems in buildings hinges in large part on their economic performance relative to conventional heating and cooling systems. Economic evaluations and comparisons of alternative solar and conventional energy systems will be required by research analysts, by builders, homeowners, architects, lenders, manufacturers of solar energy equipment, government policy makers, and others in order to determine the economic merits of solar energy systems. To make these evaluations, reliable and consistent procedures are needed for the collection and analysis of economic costs and benefits associated with the various systems.

1.2 Purpose

The purpose of this paper is (1) to explain and illustrate with simple, but realistic examples some basic concepts and techniques for evaluating and comparing the economic efficiency of solar and conventional heating and cooling systems for buildings, (2) to assess the appropriateness of alternative assumptions and procedures which might be employed in the economic evaluation of alternative systems, and (3) to set forth the necessary economic conditions for determining efficient tradeoffs in system/building design.

The paper is intended primarily as a resource document for researchers and analysts who are concerned with the design and evaluation of solar heating and cooling systems for buildings. Although not definitive in all areas, it aims at providing sufficient background to facilitate later development of a simplified, consumer-oriented handbook to assist homeowners, builders, and others in evaluating the private costs of alternative heating and cooling systems.

1.3 Scope and Organization

The economic evaluation of solar heating and cooling systems, examined in Section 2, is approached from the standpoint of life-cycle cost analysis. Solar and conventional HVAC (Heating, Ventilating, and Air Conditioning) systems are viewed as alternative means of maintaining a dwelling at a specified temperature, and the focus is on the acquisition, maintenance, and operation costs over the life of a solar heating and cooling system as compared with a counterpart conventional system. Other possible cost differences in solar and conventional systems, such as in their pollution effects, are recognized as important, but are not considered here. The relevant cost elements for life-cycle costing are set forth, the minimum data requirements are identified, and several alternative approaches (equal in their results) are described for evaluating the efficiency of comparable HVAC systems. These approaches include life-cycle cost analysis and benefit-cost analysis. The appropriate assumptions regarding period of analysis, discount rate, and escalation of costs over time are discussed, and a tentative set of assumptions is set forth. The focus is on solar heating and cooling of new buildings, but the same techniques and procedures would also apply to the application of solar energy systems to existing buildings. The economics of applying solar systems to both residential and commercial buildings is considered.

Section 3 of the paper is concerned with the problem of making cost-effective tradeoffs at the design state--including tradeoffs among components of solar heating and cooling systems, such as thermal storage capacity versus collector size; tradeoffs in the proportion of heating and cooling to be provided by the solar system and by the auxiliary support system; and tradeoffs between energy conserving measures and heating and cooling loads. For purpose of illustration, the focus of this part is on the latter tradeoff, i.e., finding the economically efficient combination of building envelope and internal space conditioning system which will satisfy user heating and cooling requirements. The necessary economic conditions for achieving the efficient combination are identified.

Both Sections 2 and 3 of the paper consider costs in a life-cycle context, but their focus is different. Section 2 outlines the data and methods required for the life-cycle cost evaluation of given solar system designs. Section 3 focusses on system evaluation during the conceptual design stages of the total solar dwelling; life-cycle costing is assumed in the analysis of design alternatives for the combination of the HVAC system and the building envelope.

Section 4 summarizes the paper briefly. It also suggests areas for further research.

2. LIFE-CYCLE COST EVALUATION OF SOLAR HEATING AND COOLING SYSTEMS

This section outlines the use of life-cycle cost analysis to evaluate solar heating and cooling systems. The emphasis is on specifying the kinds of private costs which should be taken into account by owners of buildings, and the general method for deriving and comparing costs of a proposed or constructed solar energy system with a counterpart conventional system. The purpose is to develop guidelines for evaluating the economic feasibility of solar energy systems.

Before discussing the methodology of life-cycle costing, let us consider briefly the rationale for focusing on the life-cycle costs of solar energy systems to consumers, i.e., private costs.

2.1 Rationale for a Life-Cycle Evaluation of Private Costs

As was noted in the Introduction, this analysis is confined to an examination of direct outlays by the purchaser of a heating and cooling system for a building; social costs or social benefits from external diseconomies or economies, such as air pollution from fossil fuels, are not included. The reason for limiting the examination to private costs is that they are the relevant factor in the widespread adoption or rejection of solar energy systems for heating and cooling residences and commercial buildings. Private decision making, such as selection of an HVAC system, does not generally take into account all social costs or social benefits.

The reason for emphasizing costs, as opposed to comfort benefits,[1] in this paper is twofold: (1) Most importantly, there will probably not be important differences in the comfort performance (i.e., in private benefits) of alternative heating and cooling systems, in most cases. For systems which are about equal in their comfort performance and in their satisfaction of constraints, a comparison of costs alone is adequate to determine the more efficient system. (2) Secondly, the differences which are perceived in benefits may be difficult to quantify. For example, it would be difficult to place a value on the "novelty appeal" which a consumer might derive from having a non-conventional system. In any case, use of a life-cycle cost model does not preclude subjective evaluation of nonquantifiable benefit attributes. Furthermore, differences in system performance can be handled in a cost formulation simply by treating positive differences as negative costs.[2] (This technique is used in Section 2.5.4.1 to account for differences in rental income for solar-equipped versus conventionally-equipped commercial buildings.) In most cases a comparison of explicit costs of alternative systems will be the easiest evaluation approach and will probably suffice in view of a number of other inaccuracies.

---

[1]Section 2.6 presents a benefit-cost comparison of alternatives, but "benefits" is there defined as the fuel cost savings from a solar energy system, rather than as a difference in the level of comfort performance provided by the alternative systems.

[2]In evaluating HVAC systems, the distinction between system costs and benefits is not always clear, and often differences may be treated as either. For example, greater reliability of HVAC system A compared with system B may be treated in an economic analysis as a benefit for A, as a negative cost for A, or as a positive cost for system B.

Now let us consider the usefulness of a life-cycle cost evaluation. Apart from novelty appeal or the desire of building operators to insure satisfaction of their heating and cooling needs in face of the vagaries of the international oil market, there are at least three conditions under which solar energy systems would tend to be used: (1) if solar systems are economically competitive with conventional systems under competitive market conditions; (2) if a system of governmental incentives for solar energy (or penalties for conventional energy) made solar more attractive than conventional energy systems; (3) if "energy moratoriums" were imposed, prohibiting or limiting the use of nonrenewable energy sources, such that construction of new buildings would require provision of a nondepletable energy source such as solar energy. These latter two situations might arise for reasons of national defense, environmental considerations, or from the actual unavailability of fossil fuels. In any of these cases--free market or the constrained market situations--economic efficiency requires adoption of the least-cost system available which will satisfy constraints and heating and cooling demands. Thus, in any of these cases, an important attribute of a solar energy system is its life-cycle cost. In the competitive market context, the critical factor is the life-cycle costs of solar systems relative to conventional systems; whereas, in the constrained market context, the relevant life-cycle cost comparison might well be among alternative solar energy technologies.

It may be argued, however, that life-cycle costs have not been a guiding factor in building decisions in the competitive market--that architects, builders, buyers, and the financial community have been generally more concerned with the first cost, the size of the down payment, and the monthly mortgage payment than with the total effective costs of a residence to its owner over its life. And to a large extent, this observation is valid. But lack of attention in the past does not change the fact that the life-time cost effectiveness of alternative building systems will probably become a guiding factor in investment and purchasing decisions of the future. There are indications of more attention to residential fuel and utilities costs as these costs have risen.

2.2 Life-Cycle Cost Analysis: An Overview

The technique of life-cycle cost analysis considers total relevant costs over the life of a system, including costs of acquisition, maintenance, operation, and where applicable, disposal. It is a useful approach both in the comparative analysis of design or ownership alternatives and in the collection of data for purpose of future analysis. The life-cycle cost concept is an appropriate approach to cost analysis in both the Federal and private sectors.

The major steps in performing life-cycle cost analysis are the following:

1. Specification of Objectives and Constraints

2. Identification of Alternative Solutions

3. Identification of Relevant Cost Items for Each Alternative

4. Determination of Amounts and Timing of Cash Flows

5. Calculation of Life-Cycle Costs

6. Comparison of Costs for Alternatives

This list shall be used to guide the discussion of life-cycle cost analysis of solar energy systems; each step will be treated in turn.

2.3 Objectives, Constraints, and Alternative Solutions

The relevant life-cycle cost objective is to achieve a desired level of thermal comfort in the home, in terms of temperature, humidity, and other related attributes, at lowest cost, while also meeting possible constraints, such as safety or aesthetics.

There are a number of possible alternative approaches to this objective. They would include use of conventional heating and/or cooling systems, such as a natural gas system, a propane system, an oil system, an electric resistance system, or an electric heat pump system. The alternatives would also include solar energy systems of varying design, as well as energy conservation investments to reduce heat loss or gain to the residence and thereby reduce the required capacity and level of operation of the heating and cooling system. Generally, the alternatives are different combinations of solar and conventional energy systems and energy conservation. (See Section 3 for a discussion of trade-offs among alternatives.)

A given solar heating and cooling system would be evaluated against these other alternatives to determine the least-cost means of accomplishing the comfort objective. The alternatives may differ substantially in their comparative costs, but the direction and size of the differences over the life of the building or the period of use may not be apparent without an explicit cost analysis.

2.4 Relevant Costs

As noted above, life-cycle costing takes into account costs over the life of the system, rather than first costs only. Thus, for the purchaser, life-cycle costing of solar and conventional heating and cooling systems requires assessment of the following kinds of costs: (1) system acquisition costs, including search costs, purchase prices, delivery costs, and installation costs; (2) system repair and replacement costs; (3) maintenance costs; (4) operating costs, comprising mainly energy cost; (5) insurance; (6) taxes, and (7) salvage values, net of removal and disposal costs.

These costs are required for all parts of the system being costed. The principal solar subsystems for which costs would be collected are the following: (1) solar collector, (2) thermal storage, (3) domestic hot water system, (4) air conditioning components (e.g., absorption system), (5) auxiliary energy subsystems (may include heat pump), (6) heating and cooling distribution subsystems, and (7) the control subsystem. Motors, pumps, fans, blowers, wiring, and tubing are included in these subsystems.

For purpose of comparison, cost of acquisition, repair and replacement, operation, maintenance, insurance, taxes, and salvage values would also be required for the heating and cooling units and the distribution and other applicable subsystems for the counterpart conventional system(s).

In addition, the HVAC system (solar or conventional) is an integral part of the total dwelling system, and may thereby affect costs of the building envelope and other building subsystems. For example, use of a solar energy

system may impose special structural requirements on the building envelope, such as additional roof supports to bear the weight of the collector, or special siting requirements to enable efficient operation of the solar energy system. Necessary alterations may increase usual building costs or may be cost-reducing, as in the case of a reduction in the cost of conventional roofing that results from replacing it in part by the solar collector component. Also, the optimal expenditure for certain envelope features (e.g., insulation, roof overhangs for solar shading, and storm doors and windows) may be different among different types of energy systems. Neglect of these differences in related building costs would distort the comparison of solar and conventional systems.

An additional cost, not included in the above listing of costs because of its difficulty to quantify, is the cost of future system modification, which might be undertaken, for example, for purpose of modernization or expansion. If it is an important selection criterion, differences in the relative flexibility or adaptability of alternative HVAC systems can be taken into account in system comparisons by assigning either a cost value for inflexibility or a benefit value for flexibility of one system relative to another.

A cost item included in the above listing which may warrant explanation, is "search cost." Search cost refers to the cost to the purchaser of obtaining the information necessary to consider use of a solar system. It would include the cost (in time and direct money outlay) of determining the technical suitability of a solar system for a given building, of determining the availability of solar designs, of identifying the availability of maintenance service for solar systems, etc. It is, in short, a "nuisance" cost arising from the typical builder's and consumer's lack of experience with and knowledge of solar systems. This cost may not be easily quantifiable; it is variable by purchaser; and it will tend to change over time. For most purposes, it will be sufficient to treat this cost subjectively, noting, for example, that if the costs of a solar and a conventional energy system are identical in all respects except search costs, that the solar system will be disadvantaged in this respect. As these costs are reduced by greater consumer knowledge and/or are incorporated into purchase price, it will become unnecessary to consider them separately from purchase and installation price.[1]

A life-cycle cost comparison of alternative systems may be based on total costs for each system or on the differences between systems. It is, however, the cost differences which are critical to determining efficient choices between alternatives. Cost items which are identical for the alternatives can be conveniently omitted from a comparison without changing the outcome of the analysis. For example, if the auxiliary heating system of a solar heating system is identical (or nearly identical) to the conventional system which would be used alone, omitting the first cost of the conventional system and of the auxiliary system from the life-cycle cost comparison would not matter, since the addition of an equal sum to each alternative's cost would not alter the difference between them.

---

[1] Government programs (such as the Solar Demonstration Program authorized by the Solar Heating and Cooling Demonstration Act of 1974) which increase knowledge and information regarding solar energy systems, tend to reduce search costs or at least shift them from the direct consumer to the general taxpayer. And, as manufacturers of solar energy systems provide more information through advertising, service contracts, and warranties, search costs will tend to become reflected in the supply price of solar energy systems.

Collection of total cost data (as opposed to cost differences), however, is generally preferable when the analysis is not being made "on the spot" or by an analyst directly familiar with system costs. This recommendation would apply, for example, to the analysis of government sponsored solar heating and cooling projects. The requirement of total cost data would help to prevent the omission of relevant costs, and would allow the analyst greater flexibility in the method of analysis. Furthermore, the collection of total life-cycle costs, rather than cost differences, is preferable for the purpose of developing a historical data file which may be used for other analyses.

Table 1 depicts the principal kinds of cost data which would be used to evaluate solar and conventional energy systems. At a minimum, acquisition, maintenance, replacement, and operating costs are required for each of the subsystems listed. For more extensive analysis of costs by subsystem, additional detailing of costs within subsystems would be necessary. (Insurance and tax costs are not shown in Table 1; assessment of insurance and tax impacts on costs are discussed in some detail in a later section.)

In evaluating the expected future cost effectiveness of an experimental solar system which has not yet been produced in quantity, the analyst may wish to project future costs of the system, in addition to measuring costs of the prototype system. In this case, data requirements will encompass costs of the prototype system, as well as projections of future costs for the system, based on a set of assumptions regarding technological change and production volume.

2.5 Cash Flows and Life-Cycle Costs

2.5.1 Cash Flows

After selection of the conventional alternatives to which solar heating and cooling is to be compared and identification of all the relevant costs for each alternative, the next step in the analysis is to determine the amount and timing of positive and negative cash flows associated with each alternative. The costs and their time of occurrence can be conveniently summarized by using cash flow diagrams. It is necessary to take account of the timing of cash flows because money has a time value, and, therefore, equal expenditures made at different times do not have the same value.[1]

2.5.2 Discounting of Costs

In order to compare systems, it is necessary to convert the expenditures for each system to an equivalent base. This is done by applying appropriate discounting formulas to costs to convert them all to either a present value basis or an annual cost basis. There are six basic formulas which are used to move values in time so that they may be compared on an equivalent basis with

---

[1]The time value of money reflects the opportunity for investment which will yield a real return; it is a consideration apart from inflation.

Table 1

COST DATA REQUIREMENTS

	Acquisition Cost			Replacement Cost & Salvage Value			Annual Maintenance and Repair Cost		Operating Costs (Energy Costs)		
	Purchase	Delivery	Installation	Cost, New Part	Salvage, Old Part	Time of Replacement	Labor	Materials	Energy Load	Type & Quantity of Required Energy Source	Annual Cost[a] At Current Prices
SOLAR SYSTEM											
Solar Collection Subsystem											
Thermal Storage Subsystem											
Systems Control											
Heating and/or Cooling Distribution Subsystem											
Domestic Hot Water Storage Subsystem											
Space Cooling Components											
Living Space Occupied by Subsystems											
Auxiliary Heating Units											
Building Costs											
Other, e.g., heat pump											
CONVENTIONAL SYSTEM											
Heating Unit											
Cooling Unit											
Heating and/or Cooling Distribution Subsystem											
Domestic Hot Water System											
Systems Control											
Building Costs											
Occupied Living Space											
Other											

[a]Computed for the prices of the energy sources used, e.g., price per gallon of heating oil.

values associated with other systems. These formulas are shown in Table 2, together with their standard nomenclature.[1]

Tabular solutions (called discount factors) for these formulas are available in most engineering economics textbooks[2] for a range of values of the parameters i and N. Use of these tables can greatly simplify the discounting of costs. For purpose of illustration, tabulated values for the uniform capital recovery formula and the single present value formula are shown in Table 3. Discount factors from both of these tables are used in the life-cycle cost models developed in the following section. By multiplying a given dollar value by the appropriate discount factor for a selected time and discount rate, one obtains the same solution as would be obtained by applying the corresponding discount formula to the given value. For example, with a discount rate of 10%, the present value of a \$500 cost to be incurred 10 years from now may be calculated by applying the single present worth discount formula to the future amount (i.e., $P = \$500 \frac{1}{(1 + .10)^{10}} = \$198$), or by multiplying \$500 by the single present worth discount factor for 10 years at 10% (i.e., P = \$500 (.3855) = \$198).

The discounting of costs requires selection of a discount rate. In general, the appropriate rate for discounting costs is the rate of opportunity cost to the investor; i.e., the rate of return foregone on the next best alternative investment.

Discount rates may be expressed in nominal (market) terms or in real terms. Nominal rates include an inflation factor, whereas real rates are net of inflation. Either expression may be used as long as costs to be discounted are expressed in corresponding terms. That is, if a market rate of interest is used to discount costs, future costs should be estimated to include inflationary price changes; if a real rate of interest is used to discount costs, future costs should be estimated net of inflation. To simplify the analysis and avoid unnecessary computations, a customary practice is to assume--in absence of strong evidence to the contrary--that all prices, costs, and incomes inflate or deflate at the

---

[1] Variations of these basic formulas are often used in life-cycle cost analysis. For examples, the uniform present worth formula may also be written as

$$\frac{A[1 - \frac{1}{(1 + 1)^N}];}{i}$$

and the single present worth factor may be used with a price escalation factor (e) in the numerator to calculate present value of a yearly cost (Y_o) escalating at a fixed rate, i.e.,

$$P = Y_o \sum_{j=1}^{N} \left(\frac{1 + e}{1 + i}\right)^j = Y_o \frac{(1 + e)}{(i - e)} \left(1 - \left(\frac{1 + e}{1 + i}\right)^N\right).$$

[2] See, for example, Gerald W. Smith, Engineering Economy: Analysis of Capital Expenditures, 2nd ed. (Ames, Iowa: The Iowa State University Press, 1973); and Eugene L. Grant and W. Grant Ireson, Principles of Engineering Economy, 5th ed. (New York: The Ronald Press Co., 1970).

Table 2

DISCOUNT FORMULAS

STANDARD NOMENCLATURE	USE WHEN	STANDARD NOTATION	ALGEBRAIC FORM
Single Compound Amount Formula	Given P; to find F	(SCA, i%, N)	$F=P\ (1+i)^N$
Single Present Worth Formula	Given F; to find P	(SPW, i%, N)	$P=F\ \frac{1}{(1+i)^N}$
Uniform Compound Amount Formula	Given A; to find F	(UCA, i%, N)	$F=A\ \frac{(1+i)^N-1}{i}$
Uniform Sinking Fund Formula	Given F; to find A	(USF, i%, N)	$A=F\ \frac{i}{(1+i)^N-1}$
Uniform Capital Recovery Formula	Given P; to find A	(UCR, i%, N)	$A=P\ \frac{i(1+i)^N}{(1+i)^N-1}$
Uniform Present Worth Formula	Given A; to find P	(UPW, i%, N)	$P=A\ \frac{(1+i)^N-1}{i(1+i)^N}$

Where:

P = a present sum of money.
F = a future sum of money, equivalent to P at the end of N periods of time at an interest of i.
i = an interest rate.
N = number of interest periods.
A = an end-of-period payment (or receipt) in a uniform series of payments (or receipts) over N periods at i interest rate, usually annually.

Source: Gerald W. Smith, Engineering Economy, p. 47.

Table 3

DISCOUNT FACTORS

CAPITAL RECOVERY FACTORS FOR INTEREST RATES FROM 0% TO 25%

n \ i	0%	2%	4%	6%	8%	10%	12%	15%	20%	25%
1	1.00000	1.02000	1.04000	1.06000	1.08000	1.10000	1.12000	1.15000	1.20000	1.25000
2	0.50000	0.51505	0.53020	0.54544	0.56077	0.57619	0.59170	0.61512	0.65455	0.69144
3	0.33333	0.34675	0.36035	0.37411	0.38803	0.40211	0.41635	0.43798	0.47473	0.51230
4	0.25000	0.26262	0.27549	0.28859	0.30192	0.31547	0.32923	0.35027	0.38629	0.42314
5	0.20000	0.21216	0.22463	0.23740	0.25046	0.26380	0.27741	0.29832	0.33438	0.37184
6	0.16667	0.17853	0.19076	0.20336	0.21632	0.22961	0.24323	0.26424	0.30071	0.33882
7	0.14286	0.15451	0.16661	0.17914	0.19207	0.20541	0.21912	0.24036	0.27742	0.31634
8	0.12500	0.13651	0.14853	0.16104	0.17401	0.18744	0.20130	0.22285	0.26061	0.30040
9	0.11111	0.12252	0.13449	0.14702	0.16008	0.17364	0.18768	0.20957	0.24808	0.28876
10	0.10000	0.11133	0.12329	0.13587	0.14903	0.16275	0.17698	0.19925	0.23852	0.28007
11	0.09091	0.10218	0.11415	0.12679	0.14008	0.15396	0.16842	0.19107	0.23110	0.27349
12	0.08333	0.09456	0.10655	0.11928	0.13270	0.14676	0.16144	0.18448	0.22526	0.26845
13	0.07692	0.08812	0.10014	0.11296	0.12652	0.14078	0.15568	0.17911	0.22062	0.26454
14	0.07143	0.08260	0.09467	0.10758	0.12130	0.13575	0.15087	0.17469	0.21689	0.26150
15	0.06667	0.07783	0.08994	0.10296	0.11683	0.13147	0.14682	0.17102	0.21388	0.25912
16	0.06250	0.07365	0.08582	0.09895	0.11298	0.12782	0.14339	0.16795	0.21144	0.25724
17	0.05882	0.06997	0.08220	0.09544	0.10963	0.12466	0.14046	0.16537	0.20944	0.25576
18	0.05556	0.06670	0.07899	0.09236	0.10670	0.12193	0.13794	0.16319	0.20781	0.25459
19	0.05263	0.06378	0.07614	0.08962	0.10413	0.11955	0.13576	0.16134	0.20646	0.25366
20	0.05000	0.06116	0.07358	0.08718	0.10185	0.11746	0.13388	0.15976	0.20536	0.25292
25	0.04000	0.05122	0.06401	0.07823	0.09368	0.11017	0.12750	0.15470	0.20212	0.25095
30	0.03333	0.04465	0.05783	0.07265	0.08883	0.10608	0.12414	0.15230	0.20085	0.25031
40	0.02500	0.03656	0.05052	0.06646	0.08386	0.10226	0.12130	0.15056	0.20014	0.25003
50	0.02000	0.03182	0.04655	0.06344	0.08174	0.10086	0.12042	0.15014	0.20002	0.25000
100	0.01000	0.02320	0.04081	0.06018	0.08004	0.10001	0.12000	0.15000	0.20000	0.25000
∞		0.02000	0.04000	0.06000	0.08000	0.10000	0.12000	0.15000	0.20000	0.25000

PRESENT WORTH FACTORS FOR INTEREST RATES FROM 0% TO 25%

n \ i	0%	2%	4%	6%	8%	10%	12%	15%	20%	25%
1	1.0000	0.9804	0.9615	0.9434	0.9259	0.9091	0.8929	0.8696	0.8333	0.8000
2	1.0000	0.9612	0.9246	0.8900	0.8573	0.8264	0.7972	0.7561	0.6944	0.6400
3	1.0000	0.9423	0.8890	0.8396	0.7938	0.7513	0.7118	0.6575	0.5787	0.5120
4	1.0000	0.9238	0.8548	0.7921	0.7350	0.6830	0.6355	0.5718	0.4823	0.4096
5	1.0000	0.9057	0.8219	0.7473	0.6806	0.6209	0.5674	0.4972	0.4019	0.3277
6	1.0000	0.8880	0.7903	0.7050	0.6302	0.5645	0.5066	0.4323	0.3349	0.2621
7	1.0000	0.8706	0.7599	0.6651	0.5835	0.5132	0.4523	0.3759	0.2791	0.2097
8	1.0000	0.8535	0.7307	0.6274	0.5403	0.4665	0.4039	0.3269	0.2326	0.1678
9	1.0000	0.8368	0.7026	0.5919	0.5002	0.4241	0.3606	0.2843	0.1938	0.1342
10	1.0000	0.8203	0.6756	0.5584	0.4632	0.3855	0.3220	0.2472	0.1615	0.1074
11	1.0000	0.8043	0.6496	0.5268	0.4289	0.3505	0.2875	0.2149	0.1346	0.0859
12	1.0000	0.7885	0.6246	0.4970	0.3971	0.3186	0.2567	0.1869	0.1122	0.0687
13	1.0000	0.7730	0.6006	0.4688	0.3677	0.2897	0.2292	0.1625	0.0935	0.0550
14	1.0000	0.7579	0.5775	0.4423	0.3405	0.2633	0.2046	0.1413	0.0779	0.0440
15	1.0000	0.7430	0.5553	0.4173	0.3152	0.2394	0.1827	0.1229	0.0649	0.0352
16	1.0000	0.7284	0.5339	0.3936	0.2919	0.2176	0.1631	0.1069	0.0541	0.0281
17	1.0000	0.7142	0.5134	0.3714	0.2703	0.1978	0.1456	0.0929	0.0451	0.0225
18	1.0000	0.7002	0.4936	0.3503	0.2502	0.1799	0.1300	0.0808	0.0376	0.0180
19	1.0000	0.6864	0.4746	0.3305	0.2317	0.1635	0.1161	0.0703	0.0313	0.0144
20	1.0000	0.6730	0.4564	0.3118	0.2145	0.1486	0.1037	0.0611	0.0261	0.0115
25	1.0000	0.6095	0.3751	0.2330	0.1460	0.0923	0.0588	0.0304	0.0105	0.0038
30	1.0000	0.5521	0.3083	0.1741	0.0994	0.0573	0.0334	0.0151	0.0042	0.0012
40	1.0000	0.4529	0.2083	0.0972	0.0460	0.0221	0.0107	0.0037	0.0007	0.0001
50	1.0000	0.3715	0.1407	0.0543	0.0213	0.0085	0.0035	0.0009	0.0001	...
100	1.0000	0.1380	0.0198	0.0029	0.0005	0.0001	...	...	...	...

Source: Grant, Eugene L. Principles of Engineering Economy (New York: The Ronald Press Company, 1950).

same rate, such that the changes are offsetting and need not be included in the analysis.[1] On the other hand, if price changes for specific items of cost, e.g., fuel, are expected to differ from the general level of price changes and can be forecasted with some confidence, they should be included in the analysis.

Opinion is mixed regarding the appropriate real discount rate to evaluate the cost effectiveness of building systems. The historical real rate of return on business investments has generally been estimated at about 3-4%. Agencies of the Federal Government are directed to use a 10% real rate to evaluate government investments.[2] It has been suggested that the real aftertax opportunity cost of the "typical" homeowner is quite low--as low as 1% or 2%.[3] The rate on second mortgages, as an indication of the homeowner's cost of borrowing, suggests a slightly higher real rate.

Given the uncertainty of an appropriate rate, it is suggested that the evaluation of alternatives be made with several interest rates to test the sensitivity of the analysis to the discount rate. Evaluating costs on basis of a low rate equal to, say, 2% and a high rate equal to, say, 15% in real terms, would provide a reasonable range of rates for a sensitivity check.

2.5.3 Life-Cycle Cost Models

Life-cycle costs of a solar HVAC system can be computed with either a present value or an annual cost model. Both approaches take into account the changing real value of money over time. In the present value model, all costs and salvage values are forecasted over the period of analysis and then discounted to an equivalent single cost today. In the annual cost model all costs and salvage values are forecasted over the period of analysis and then are divided into uniform annual costs by discounting. Present value costs can be easily converted to an annual cost basis, and vice versa.[4]

---

[1]But in fact, the assumption that if inflation affects costs and revenues equally it does not affect the outcome of the analysis, is valid only if the HVAC system is purchased completely with equity funds (i.e., unborrowed funds), and if the system is not depreciated for tax purposes. With debt financing of the system, the payments of the purchaser tend to be fixed in amount, and their present value declines with inflation. In the case of depreciation of the system for tax purposes, which is allowable for commercial applications, inflation causes the depreciation expenses to fall in real value, since depreciation is based on the initial purchase price of the system. In addition, there are other effects of inflation beyond these two. For evaluation of solar systems in residences, these effects are not likely to be important, but in evaluating commercial applications, they may warrant attention.

[2]Office of Management and Budget. Executive Office of the President, Circular No. A-94 (Revised), March 27, 1972.

[3]Stephen R. Petersen, Retrofitting Existing Housing for Energy Conservation: An Economic Analysis, National Bureau of Standards, Building Science Series 64, December 1974, pp. 19-21.

[4]For further general discussion of present value and annual cost models, see any of the following: Gerald W. Smith, Engineering Economy; Eugene L. Grant and W. Grant Ireson, Principles of Engineering Economy; E. Paul DeGarmo, Engineering Economy, 4th ed. (New York: The Macmillan Co., 1969).

Apart from taxes, the basic formula for computing the present value of an HVAC system can be developed by applying to the cost items from Table 1, the appropriate discounting formulas from Table 2. (The effect of taxes on costs will be deferred to Section 2.5.4.) The following formula includes terms for the basic kinds of costs, i.e., acquisition, replacement, maintenance, and operating costs, and would be suitable to compute the costs of either a solar (plus conventional auxiliary) system or a conventional system alone.

$$PV = I - \frac{S}{(1+i)^N} + \sum_{j=1}^{N} \frac{(R_j - \overline{S}_j)}{(1+i)^j} + M \frac{(1+i)^N - 1}{i(1+i)^N} + F_o \sum_{j=1}^{N} \left(\frac{1+e_o}{1+i}\right)^j + F_1 \sum_{j=1}^{N} \left(\frac{1+e_1}{1+i}\right)^j + B + Q \qquad (1)$$

where

PV = present value cost of the HVAC system over period N,

I = initial investment costs, including costs of acquisition, delivery, and installation of the heating and cooling system(s),

S = remaining value of the HVAC system(s) at the end of the period of analysis,

i = annual discount rate in real terms,

N = period of analysis in years, (may be the life of the building or a shorter designated period),

R_j = Replacement and repair costs in year j at present prices, including costs or replacing or repairing any part of the system,

$\overline{S}_j$ = salvage value in year j, where $j < N$, at present prices, of replaced parts,

$R_j - \overline{S}_j$ = net replacement and repair costs in year j,

M = estimated annual maintenance cost at present prices, assumed here to be constant over the life of the system. (Alternatively, these costs might be assumed variable from year to year, in which case they could be included in the repair and replacement term; or they might be assumed to escalate at a constant rate or amount over time, in which case they could be treated as fuel costs are treated above or discounted by use of gradient series interest formulas[1], respectively.)

---

[1]Interest computations for periodic sums changing by an equal amount each period or by an equal rate each period are explained in Gerald W. Grant, Engineering Economy, pp. 47-52.

F = estimated annual energy cost at present prices; subscripts indicate different sources of energy, e.g., F_o might indicate #2 heating oil and F_1 might indicate electricity cost.

e = annual rate of change in real price of energy, where subscripts indicate different sources of energy,

B = initial investment costs for HVAC - related building modifications (if modifications are cost-reducing, B will be subtracted from costs).

Q = value of building space occupied by HVAC system components, evaluated as building cost/sq. ft. x number sq. ft. occupied.

Investment costs are entered in the equation without discounting, because these costs, as first costs, are already in present value terms. The remaining value (salvage) of the system when use has terminated or when the defined period of analysis has ended, is converted to present value by use of the single present worth formula, and is deducted from investment cost because it represents investment costs not actually incurred. Cost of replacing parts of the system, net of the salvage value of the old parts, are discounted from the year they are expected to be incurred to present value, summed, and added to other costs. Annual maintenance and repair costs might generally be assumed constant in real terms, and, if so, would be discounted to present value by use of the uniform present value formula.

Two terms are included for energy costs to indicate that several sources of energy of varying price might be used. The annual expenditure on each energy source should be escalated if real increases in price are expected. The escalated annual costs are then discounted to present value and summed. The last two terms, cost of building modifications and cost of building space occupied, are incurred initially, and, therefore, are already in present value equivalents.

The basic cost elements in the annual cost formula, equation (2), are identical to those in the present value equation. The only difference is in the discounting procedures. There are several ways to formulate the annual cost equation. One way is simply to apply a capital recovery factor to the present value costs as expressed in equation (1) to convert them to an equivalent uniform stream. This is essentially what has been done in the following annual cost model for computing life-cycle costs of an HVAC system, with the exception that annual maintenance cost, M, is entered directly, without discounting, since it is already in appropriate form.

$$AC = \left[I - \frac{S}{(1+i)^N} + \sum_{j=1}^{N} \frac{(R_j - \overline{S}_j)}{(1+i)^j} + F_o \sum_{j=1}^{N} \left(\frac{1+e_o}{1+i} \right)^j + F_1 \sum_{j=1}^{N} \left(\frac{1+e_1}{1+i} \right)^j + B + Q \right] \cdot \left[\frac{i(1+i)^N}{(1+i)^N - 1} \right] + M, \qquad (2)$$

where the variables are as defined previously.

Generally, in the cost analysis of building subsystems, the present value of costs would be computed for the life of the building; i.e., N would be defined as building life (usually 30 to 40 years or greater). Alternatively, N may be defined as "the period of analysis," which may be equal to, or much shorter than building life, say 20 years. Reasons for limiting the period of analysis to a shorter period than building life might be that forecasts of energy availability and conditions in the housing market become increasingly uncertain at farther points in time, or that intended use of the building is limited and resale uncertain. Other things equal, if a very low discount rate were used, the results would tend to be quite sensitive to whether the choice of periods were relatively short or long; but if a relatively high discount rate were used, the results would be much less sensitive to the length of the period.

2.5.4 Treatment of Taxes, Insurance, and Governmental Incentives

Thus far, only the more obvious elements of life-cycle costs have been treated, the effects of various institutional arrangements which may alter the effective costs of solar energy systems to the purchaser have not yet been taken into account. Taxation, for example is one institutional effect which frequently alters the cost or profitability of investment decisions. Insurance is another. In addition, state, local, or Federal programs of incentives or penalties to encourage or discourage respectively the choice of certain HVAC systems may change life-cycle costs.

Since widespread adoption of solar energy systems in buildings depends upon the effective dollar costs to the owners and users, these effects should be considered. The remainder of Section 2.5.4 discusses how the life-cycle cost formulas given earlier, equations (1) and (2) could be modified or expanded to account for the effects of taxation, insurance, and special government programs on the life-cycle costs of owning a building. It also assesses in general terms the probable direction of impact of taxation on an owner's costs.

2.5.4.1 Taxes

The effect of taxation on costs of a solar system to the owner may be considered for two main cases: (1) for the owner-occupied solar residence, and (2) for the rental solar residence or other solar commercial building. For both cases, taxes impact on costs in several ways, in some instances raising and in other instances lowering life-cycle costs of the solar HVAC system relative to its conventional counterpart. Let us examine the two cases in turn for tax implications.

For the owner-occupied residence, the primary tax effects are from the property tax and, indirectly, from the income tax. The particular effect of either of these taxes could be expected to vary considerably among individual solar residences, depending upon local property tax rates, property assessment practices, and the income tax bracket of the homeowner. The focus here is on the nature of the effects and on the method of including them in the life-cycle cost analysis.

The property tax, which is levied as a percentage of a share of the market value of a building, would tend to raise the life-cycle costs of a solar HVAC system relative to a conventional system. Life-cycle costs would be raised because, other things equal, the greater first cost of solar HVAC equipment would be reflected in a higher market value for the residence, and, hence in a larger assessed value for the solar residence than for a conventional residence with its lower first-cost HVAC system. Thus, the capital intensiveness of an HVAC system influences the amount of property taxes levied on a residence, and thereby, alters the life-cycle cost of the HVAC system.

As a simple example, let us compare the property tax on a $60,000 solar residence, of which $8,000 is attributable to additional cost of the solar HVAC system, with a counterpart conventionally heated and cooled home valued at $52,000 (i.e., $60,000 - $8,000). Given a typical tax rate of 4.50% of 50% of market value,[1] the $60,000 solar residence would be assessed at $30,000 and taxed $1,350. The counterpart conventional home would be assessed $26,000 (i.e., 50% of $60,000 - $8,000) and taxed $1,170. For purpose of illustration further assume a real discount rate of 2%, a constant real property value (including a constant real value for the solar system with replacements made as needed), and a constant property tax rate over a 20 year period of evaluation, (the assumption of a constant real property value and a constant property tax rate means that even though the nominal, or market, property value changes, the yearly property tax remains constant in terms of present prices.) Over 20 years, the property tax on the solar residence would amount in present value terms to $22,072 (i.e., $\$1{,}350\ \frac{(1 + .02)^{20} - 1}{.02\ (1 + .02)^{20}} = \$22{,}072$); the present value of the property tax on the conventional residence would amount to $19,130 (i.e., $\$1{,}170\ \frac{(1 + .02)^{20} - 1}{.02\ (1 + .02)^{20}} = \$19{,}130$). Thus, the effect of the property tax in the above example is to raise the life-cycle cost of the solar residence relative to the conventional system by nearly $3,000. This simple illustration suggests that the property tax provides a disincentive for choosing solar HVAC systems.

In the example, a constant real property value was assumed for ease of illustration. This assumption implies no real depreciation of the system over time during which necessary replacement of parts is made; i.e., the salvage value, in real terms, is assumed equal to the original first cost. In some cases this assumption might be reasonable. An alternative assumption is that the HVAC system (with parts replacements) depreciates in real terms from the time of purchase, such that little or no real value remains after, say, 20 years.

---

[1]From sample census data for single-family houses in a number of U.S. cities, assessment values appear to range from about 8% to 93% of market value; nominal property tax rates from a low of about 1% to more than 25%; and the effective tax rate from only about 1% to 4% of market value. A typical property tax might be 4.50% of 50% of the market value, i.e., an effective rate of 2.25% of the market value of a residence.

A general expression of the present value of property taxes attributable to the HVAC system (PV_t) is

$$PV_t = \sum_{j=1}^{N} \frac{(t \cdot G_j)}{(1 + i)^j}, \qquad (3)$$

where

t = the property tax rate,

G_j = the assessed value of the HVAC system in year j, in present dollars. This formula would cover both the case of a constant real assessed value for the HVAC system and the case of changing real assessed values over time.

To account for the property tax effect in the present value formula equation (1), the above term would simply be added to equation (1). In the case of the annual cost formula, it would be necessary first to convert the present value to an annual cost by applying the capital recovery discount formula; that is,

$$AC_t = \sum_{j=1}^{N} \frac{(t \cdot G_j)}{(1 + i)^j} \left[\frac{i(1 + i)^N}{(1 + i)^N - 1}\right]. \qquad (4)$$

The above term would be added to equation (2).

The income tax, in contrast to the property tax, would tend to reduce the life-cycle cost of a solar vis-a-vis conventional residence in two ways. For one thing, the homeowner is able to deduct his mortgage interest payments from taxable income. The higher first cost of the solar HVAC system, by increasing the size of the mortgage to be amortized, raises interest payments; the higher interest payments can then be deducted from income for purpose of computing income tax. The value of the tax deduction to the homeowner depends on his personal income tax bracket. Consider, for purpose of illustration, the case of a solar HVAC system whose first cost of, say, $8,000 comprises part of the homeowner's mortgage. With a 10% market rate of interest on the residential mortgage, the $8,000 amortized over 20 years would add approximately $940 per year to the mortgage payment. (For simplicity let us assume yearly mortgage payments rather than monthly payments.) The addition to the yearly payment is fixed at $940 over the 20 years, and interest comprises a declining portion of the payment over time. In the first year, interest amounts to $8000 (i.e., $8,000 x .10 = $800), and the principal is reduced by $140 (i.e., $940 - $800 = $140). In the second year interest is $786 [i.e., ($8,000 - $140) x .10 = $786], and the principal is reduced by $154 (i.e., $940 - $786 = $154); etc. Thus in the first year, the solar system would result in additional interest deductions from taxable income of $800, and, if the homeowner is in a 25% income tax racket, the end-of-year value of the deduction would be $200 (i.e., $800 x .25 = $200). At a 2% real discount rate[1] the present value of this savings would be $196 (i.e., $\frac{\$200}{(1 + .02)^1} = \$196$). In the second year, the present value return from the income tax deduction would be $188 (i.e.,

---

[1]The use of a 10% market rate of interest and a 2% real discount rate implies an inflation rate of approximately 8% annually.

$\frac{\$786 \times .25}{(1 + .02)^2} = \$188$). Over the full twenty years, the present value of the interest deductions (PV_I) would be calculated as

$$PV_I = \sum_{j=1}^{N} \frac{\bar{t}\,(L_j \cdot m)}{(1 + i)^j}, \qquad (5)$$

where

$\bar{t}$ = personal income tax rate

L_j = the additional mortgage loan principal outstanding in period j, i.e., that part associated with the HVAC system

m = the market rate of interest on the mortgage.

(During a period of price change, it would be necessary to apply a price index to the amount of the tax deductions to convert them to present prices. This conversion of current dollars to real terms would be necessary because the tax deductions are fixed, and do not reflect changing prices.) The above term would be subtracted from equation (1) to account for the effect of tax deductions of interest on the present value of the homeowner's life-cycle costs.

To account for this tax effect in the annual cost formula, equation (2), it is necessary to convert the above present value expression to an annual cost, i.e.,

$$AC_I = \sum_{j=1}^{N} \frac{\bar{t}\,(L_j \cdot m)}{(1 + i)^j} \cdot \left[\frac{i\,(1 + i)^N}{(1 + i)^N - 1}\right]. \qquad (6)$$

This term would then be subtracted from equation (2).

In addition to the deduction of interest payments, the homeowner would also be able to deduct payments of property taxes from taxable income. Thus, a part of the increase in life-cycle cost of a solar HVAC system resulting from higher property tax on the solar residence would be offset by related income tax deductions. The present value of the property tax deduction (PV_p) can be calculated as

$$PV_p = \sum_{j=1}^{N} \frac{\bar{t}\,(t \cdot G_j)}{(1 + i)^j}, \qquad (7)$$

where

$\bar{t}$ = the personal income tax rate,

t = the property tax rate,

G = the assessed value of the HVAC system in year j, in present dollars.

This term would be subtracted from equation (1) to take into account the effect of income tax deductions of property taxes on present-value costs. The annual cost formula would be adjusted for this tax effect by converting the net present value to the annual cost equivalent (as was done in the preceding case) and subtracting this from equation (2).

Alternatively, the net present value effect of the property tax and the related income tax deductions (PV_{pt}) could be included in the present value formula by adding the following single term which can be derived from equations (3) and (7).

$$PV_{pt} = (1 - \bar{t}) \sum_{j=1}^{N} \frac{(t \cdot G_j)}{(1 + i)^j} . \tag{8}$$

In conclusion, property taxes tend to increase the homeowner's cost for a solar HVAC relative to a counterpart conventional system due to the greater capital-intensiveness of the typical solar system. Income tax effects on the other hand, tend to reduce the relative costs of the typical solar system, principally due to deduction from the homeowner's taxable income of interest payments which are larger for more capital-intensive systems. With these tax effects included (and without simplifying the equation), the formula to derive present-value life-cycle costs of an HVAC system to the homeowner is the following:

$$PV_o = \underbrace{I}_{\text{Acquisition}} + \underbrace{B}_{\text{Bldg. Modif.}} + \underbrace{Q}_{\text{Occup. Space}} + \underbrace{\sum_{j=1}^{N} \frac{(R_j - S_j)}{(1 + i)^j}}_{\text{Replacements}} + \underbrace{M \frac{(1 + i)^N - 1}{i(1 + i)^N}}_{\text{Maintenance \& Repairs}} + \underbrace{F \sum_{j=1}^{N} \left(\frac{1 + e}{1 + i}\right)^j}_{\text{Energy Costs}}$$

$$+ \underbrace{(1 - \bar{t}) \sum_{j=1}^{N} \frac{(t \cdot G_j)}{(1 + i)^j}}_{\text{Property Tax, Net of Tax Deduction}} - \underbrace{\sum_{j=1}^{N} \frac{\bar{t}\,(L_j \cdot M)}{(1 + i)^j}}_{\text{Tax Deduction of Mortgage Interest Payments}} , \tag{9}$$

where PV_o = present value cost to the homeowner of an HVAC system over period N, with tax effects included, and other terms are as previously defined.

Let us now consider the tax effects on the life-cycle cost of a commercial building equipped with a solar HVAC system, as compared with a commercial building equipped with a conventional HVAC system. In the case of commercial use of solar systems the previously described property tax and income tax effects would also apply.[1] There are in addition, other income tax deductible expenses to consider in evaluating the commerically used system, such as depreciation deductions and deductions of operation and maintenance expenses. Also, after-tax rental income of commercial buildings may be influenced by the choice between solar and conventional HVAC systems, and therefore, may need to be considered.

The larger capitalized value of the solar HVAC system would result in increased deductions of depreciation from taxable income. For example, using a straight-line method of depreciation and assuming a first cost of $8,000 for the solar system, a 20 year life, and no salvage value, the annual depreciation

---

[1] The institutional treatment of property tax and interest charges would be somewhat different for commercial buildings than for owner-occupied houses in that these items of cost would be deductible as business expenses. The effect on costs, however, would be described by the same mathematical expressions as developed above.

would be \$400. The present value to the building owner of the \$400 depreciation in a given year may be found by applying his income tax rate to the \$400, and discounting that amount to the present.[1] Alternatively, a depreciation method might be used which does not yield equal yearly amounts in either real or nominal terms (e.g., a declining balance method). A general expression of the present value of the tax deduction resulting from depreciation (PV_D) is

$$PV_D = \sum_{j=1}^{N} \frac{(D_j \cdot \bar{t})}{(1 + i)^j}, \tag{10}$$

where

D = depreciation in year j, in present dollars,

$\bar{t}$ = building owner's income tax rate.

This term would be subtracted from equation (1), reducing the present value cost of the solar HVAC system. For the annual cost equation, the above expression would be converted to an annual basis, i.e.,

$$AC_D = \sum_{j=1}^{N} \frac{(D_j \cdot \bar{t})}{(1 + i)^j} \left[\frac{i\ (1 + i)^N}{(1 + i)^N - 1}\right], \tag{11}$$

and this value would be subtracted from equation (2).

On the other hand, a solar HVAC system would generally involve lower operating (fuel) costs than its conventional counterpart. Tax deductions for operating costs would, therefore, tend to be lower for a solar system than for its conventional counterpart.

Because of the time value of money, the present value of depreciation expenses are less than the present value of the capital expenses upon which they are based. In contrast, the present value of the deductible operating expenses are approximately equal to the corresponding operating expenses incurred. Consequently, if present value capital costs are substituted (traded off) for present value operating costs on a dollar-for-dollar before-tax basis, there will not be a corresponding dollar-for-dollar tradeoff on an after-tax basis. Rather, the present value of after-tax capital costs will increase relatively more than operating costs decline, and after-tax total costs will, therefore, rise as a result of the more capital-intensive system. Hence, the fact that operating costs are fully deductible as a current business expense, while capital costs are deductible only as a depreciation expense may in some cases bias building owners towards relatively less capital-intensive conventional HVAC systems over solar HVAC systems. In the case of systems sized for small buildings, the biasing effect will probably be inconsequential, but for large buildings, it may significantly discourage the selection of solar HVAC systems.

---

[1]For purpose of this illustration, future price change is not considered, and the yearly depreciation found by dividing the first cost of the solar HVAC system (V) by the number of years of its assumed life (n), is taken as the annual depreciation cost. In fact, however, an amount of depreciation fixed in nominal dollars would become a decreasing real amount during a period of inflation. To take account of changes in the real value of annual depreciation, it would be necessary to apply a projected price index for each future year to $\frac{V}{n}$, thereby converting depreciation to present dollar terms.

Thus far the cost evaluation formulations have been based on the assumption of equal private benefits for the solar HVAC system and the conventional counterpart to which it is compared. However, in the case of some rental properties, particularly low-density rental residences, it may be necessary to take into account possible differences in rental revenue. Where conventionally provided utilities are paid by the tenant, rental revenue could be expected, other things equal, to be higher on a solar residence than on a comparable conventionally-equipped residence. That is, the owner of a solar rental residence would incur the costs of solar equipment that would be reflected in higher rent but lower utility bills to the tenant. The owner of the rental solar residence would require higher rental payments to cover his higher capital costs. Assuming other things equal and a well functioning market, tenants should be willing to pay an additional amount of rent up to the amount of the additional utilities outlay which they would incur in a counterpart conventionally-equipped residence (i.e., an amount sufficient to equalize the life-cycle costs to the tenants of counterpart solar and conventional rental units).

Different amounts of benefits (i.e., rental income) for buildings equipped with different HVAC systems means that benefits of the alternative systems are unequal. To compare the alternative systems, differences in their benefits as well as in their costs should be evaluated. Inequalities in benefits can be treated in the present value and annual cost equations as negative costs, by entering, in this case, as a negative cost any additional after-tax rental income generated by the rental solar residence over the conventional counterpart. Annual after-tax rental income (Y_T) would be expressed as

$$Y_T = (1 - \bar{t})\ Y, \qquad (12)$$

where

Y = additional annual gross rental revenue for a solar residence over its counterpart conventional residence, and

$1 - \bar{t}$ = the factor applied to obtain after-tax income.

This additional amount of annual income, i.e., $(1 - \bar{t})\ Y$, would be subtracted from equation (2) to adjust the annual cost formula. To adjust the present value formula, the term would be converted to present value equivalence and subtracted from the equation; i.e., the expression

$$PV_{Y_t} = (1 - \bar{t})\ Y \left[\frac{(1 + i)^N - 1}{i\ (1 + i)^N}\right] \qquad (13)$$

would be subtracted from equation (1).

Taking into account tax effects, the formula to derive present-value life-cycle costs of an HVAC system to the owner of a commercial property (PV_R) is the following:

$$
PV_R = \overbrace{I}^{\text{Acquisition}} + \overbrace{B}^{\text{Bldg. Modif.}} + \overbrace{Q}^{\text{Occup. Space}} + \overbrace{\sum_{j=1}^{N} \frac{(R_j - S_j)}{(1 + i)^j}}^{\text{Replacements}} + \overbrace{M \frac{(1 + i)^N - 1}{i\,(1 + i)^N}}^{\text{Maintenance \& Repairs}} + \overbrace{F \sum_{j=1}^{N} \left(\frac{1 + e}{1 + i}\right)^j}^{\text{Energy Costs}}
$$

$$
+ \overbrace{(1 - \bar{t}) \sum_{j=1}^{N} \frac{(t \cdot G_j)}{(1 + i)^j}}^{\text{Property Tax, Net of Tax Deduction}} - \overbrace{\sum_{j=1}^{N} \frac{\bar{t}(L_j \cdot M)}{(1 + i)^j}}^{\text{Tax Deduction of Mortgage Interest Payments}} - \overbrace{\sum_{j=1}^{N} \frac{(E_j \cdot \bar{t})}{(1 + i)^j}}^{\text{Other Tax Deductions [1]}}
$$

$$
- \overbrace{(1 - \bar{t})Y \frac{(1 + i)^N - 1}{i(1 + i)^N}}^{\text{Additional After-Tax Income}} \tag{14}
$$

2.5.4.2 Insurance

There are several reasons why insurance costs might differ for a solar-equipped residence as compared with one conventionally equipped. For one thing, the homeowner might insure his larger capital investment from damages, e.g., the solar collector from breakage by natural or human forces. For another, the fire insurance rate for a solar HVAC system may be less than that for a conventional system used alone, because of such factors as a smaller use of a fired furnace. Differences in probability of fire occurrence may become reflected in differential insurance rates.

Since insurance costs are one of the costs of operating a residence, they should be considered in the life-cycle cost analysis. In so doing, it is important to remember that insurance costs represent a tradeoff to the homeowner for incurring damage costs. It is the net cost to the homeowner of damage (I_n) which is relevant; that is, the cost of insurance (insurance premiums) plus damage losses, net of insurance reimbursements collected. The net cost in annual cost terms is as follows:

$$
I_n = I + L - C, \tag{15}
$$

where

I = annual insurance premiums,

L = annual damage loss,

C = Annual insurance reimbursements.

---

[1] E_j = Depreciation and other deductibles in year j, at present prices.

Alternately, M in both equations (1) and (2) could be redefined as net annual maintenance and repair costs, where repair costs are adjusted to account for annual insurance outlays and receipts.

2.5.4.3 Governmental Incentives

Effective life-cycle costs to owners and users of solar-equipped buildings may be further changed by governmental programs designed to encourage adoption of solar HVAC systems. These programs might offer special incentives for solar systems in the form of tax credits, low interest loans, or direct grants or subsidies to manufacturers and/or buyers of solar HVAC systems. Alternatively, incentives for solar systems might be provided in the form of penalties applied to conventional HVAC systems, such as by taxing conventional HVAC systems more severely than solar systems.[1] In either case, if the comparative cost to the homeowner is altered by special programs, the cost evaluation should reflect the induced changes.

The method of treating the cost effects of such programs would vary. Subsidies to producers of solar systems, for example, might be reflected in the lower purchase price of the systems, and no additional expression need be introduced into the life-cycle cost model in order to assess this effect. On the other hand, a subsidy to the purchaser of a solar HVAC system, say, in the form of a low interest loan for the purchase of a solar home, might require specific evaluation of the interest subsidy, including income tax effects.

Some programs intended to provide incentives for purchase of solar energy systems may do this by reducing previously existing disincentives for solar energy. For example, some states and localities are exempting from the property tax some part of the first cost of a solar HVAC system (e.g., in Indiana, a 1974 law requires that county property assessors exempt up to $2,000 of the cost of a solar heating or cooling system installed in a residential or commercial building form the real property tax).

A comprehensive evaluation of the effects of government incentives on life-cycle costs of solar versus conventional HVAC systems is beyond the scope of this paper. However, a general treatment of the expected impact of alternative incentive techniques would be a useful background to the cost evaluation of specific solar residences and commercial buildings.

2.6 Comparison of Alternatives

To compare solar and conventional systems, several approaches can be used. One approach is to calculate the life-cycle costs of each system with either a present value or annual cost formula as depicted by equations (1) and (2). Cost differences between systems may then be found simply by subtracting the life-cycle cost equation for one system from that of another, term by term if desired. In this way, the system with the lowest life-cycle costs can be identified.

---

[1] This discussion of governmental incentives is not intended to recommend incentives; the purpose is only to identify all the factors which should be considered in a comprehensive analysis of solar HVAC systems.

There are a number of additional methods for analyzing investment decisions, such as benefit-cost analysis, internal rate of return or yield method, payback method, and return-on-investment method.[1] These methods are used to analyze conventional investment problems, which involve both cash outlays and cash inflows.

For heating and cooling systems, cash flows are generally negative. It is possible, however, to structure this evaluation problem in conventional investment terms, by focusing on the incremental investment cost which is typically required for a solar HVAC system over a conventional system, and on the reduction in energy costs resulting from the higher investment cost. Converting the problem into one which involves costs and savings (reductions in costs) allows us to use these additional methods of analysis.

The payback method and the return-on-investment method are popular methods of analysis because they are easily calculated and readily understandable. However, they characteristically have two important weaknesses which may seriously distort the evaluation, and, therefore, they are not recommended as principal methods for evaluating alternative systems. First, they do not (as usually calculated) take into account the timing of cash flows. Second, they do not take into account the magnitude of total benefits (i.e., total energy cost reductions over the period of analysis), or total costs. The payback method ignores benefits which accrue after the payback date; the return-on-investment method focuses on average returns. These methods, nevertheless, may be useful in providing supportive evaluation information.

Neither is the internal rate of return, or yield, method recommended, although it usually gives the correct solution. Shortcomings of the internal rate of return method are that (1) it is more cumbersome to calculate, and (2) indeterminant solutions may result under some circumstances.

A net present value benefits method is in most cases a reliable approach to evaluating a capital investment. This technique can be used to convert a problem in which all cash flows are negative into a conventional investment problem involving cash outlays and cash benefits. This is a suitable method for evaluating solar HVAC systems vis-a-vis conventional systems, because solar systems generally involve a larger investment cost than conventional systems, but give rise to less operating costs than conventional systems. That is, they result in savings (i.e., benefits) in the form of reduced energy costs.

Using this approach, the costs are defined as the present value[2] of the extra costs of owning and maintaining a solar system and the benefits as the present value of savings in energy costs for a solar system as compared with a conventional system.[3] The difference between these costs and benefits, i.e., net benefits, is the measure of the efficiency of the investment in a solar system as compared with a conventional system.

---

[1]For comprehensive descriptions of these techniques, see Gerald W. Smith, Engineering Economy, pp. 87-190, and Eugene L. Grant and W. Grant Ireson, Principles of Engineering Economy, pp. 66-147.

[2]The analysis could be stated in terms of uniform annual value rather than present value, in which case net annual benefits would be calculated.

[3]As used here this approach assumes that the benefits of the alternative systems in terms of heating and cooling performances are equal, and that the systems will differ only in their costs.

Positive net benefits indicate that the solar system is more efficient, i.e., more cost effective, than the conventional system. Negative net benefits indicate that the conventional system is more efficient.[1]

The net benefits of the extra investment required for the solar system are calculated as follows:

$$B_n = (F_c - F_s) - (C_s - C_c), \tag{16}$$

where

B_n = Net benefits of a solar system as compared with a conventional system, in present value terms over N years,

F_c = Energy cost for the conventional system, in present value terms over N years,

F_s = Energy cost for the solar system, in present value terms over N years,

$F_c - F_s$ = Energy cost savings in present value terms over N years,

C_s = Capital and maintenance cost for the solar system in present value terms over N years,

C_c = Capital and maintenance cost for the conventional system in present value terms over N years,

$C_s - C_c$ = Extra investment for the solar system in present value terms over N years.

Table 4, which is in 5 parts (A through E), is an illustration of a benefit-cost comparison of a solar heating system and a conventional heating system. It is based on hypothetical data, for a specific set of assumptions. (Note that the purpose of this example is to demonstrate the methods of evaluation, and not to present evaluation results of actual systems. Therefore, undue attention should not be given to the numerical outcome.)

Part A of Table 4 shows the general assumptions upon which the example is based, including the heating load, type of systems costed, period of analysis, discount rate, and two rates of cost escalation. Part B shows the <u>additional costs</u> of acquisition and maintenance for the solar system over the counterpart conventional system. It is assumed that the solar system requires a conventional backup (auxiliary) system of size and with acquisition and maintenance costs equal to the conventional system used alone. This assumption reflects the need to ensure adequate heating during long periods of cloudy weather when the solar system is unable to contribute to heating requirements.

Part C shows the assumed annual energy costs for both systems and the annual energy cost savings (benefits) for the solar system in present prices without price escalation. Energy costs for the solar system ($194) consist of electricity costs to power motors and pumps ($20) both for the solar components and the

---

[1] Alternatively a benefits-to-costs ratio could be computed, in which a ratio greater than 1 indicates that the extra investment costs of the solar system is economically worthwhile, and a ratio less than 1 indicates that the investment in the solar system is not efficient.

Table 4

PART A

COST ANALYSIS: A HYPOTHETICAL EXAMPLE[a]

General Assumptions

° AVERAGE ANNUAL TOTAL HEATING LOAD: 84,000,000 BTU'S

° SOLAR SYSTEM: SOLAR → 60% LOAD
CONVENTIONAL AUXILIARY → 40% LOAD
EFFICIENCY, AUXILIARY → 55%
FUEL → #2 HEATING OIL

° CONVENTIONAL SYSTEM: 100% LOAD
EFFICIENCY → 60%
FUEL → #2 HEATING OIL

° PERIOD OF ANALYSIS: 20 YEARS

° DISCOUNT RATE: 2% REAL RATE

° COST ESCALATION: (1) NO REAL CHANGE
(2) 4% REAL INCREASE IN FUEL COSTS ONLY, COMPOUNDED ANNUALLY

---

[a]In view of the presently accepted practice for building technology in this country, common U.S. units of measurement have been used in this example. (To convert Btu/hr, ft^2 to W/m^2, multiply by 3.152; to convert ft^2 to m^2, multiply by 9.290×10^{-2}; to convert lbm. to Kg, multiply by 4.535×10^{-1}.) The reader interested in using the system of SI units more extensively is referred to American Society for Testing and Materials, Metric Practice Guide, ASTM No. #380-72, 1972.

Table 4

PART B

Assumed Additional Capital (Including Installation) and Maintenance Costs for Solar System

	CAPITAL COSTS	MAINTENANCE COSTS
COLLECTOR	\$7,200/20 YRS. (800 FT.^{2} @ \$9.00 PER FT.^{2})[b]	\$25/5 YRS.[a]
THERMAL STORAGE TANK	\$400/20 YRS. (8000 LBM. FLUID)	\$25/YR.
PIPES, FITTINGS	\$200/20 YRS.	
MOTORS, PUMPS	\$200/10 YRS.	
HEAT EXCHANGER	\$100/20 YRS.	
SYSTEMS' CONTROL	\$150/20 YRS.	
BUILDING MODIFICATIONS		
ROOF	\$100/20 YRS.	
INSULATION	\$75/20 YRS.	
BASEMENT SPACE	\$125 (25 FT.^{2} @ \$5.00 PER FT.^{2})	
AUXILIARY HEATING UNIT	(SAME AS CONVENTIONAL SYSTEM)	

\$8,550	ADDITIONAL FIRST COST FOR THE SOLAR SYSTEM
\$8,714	PRESENT VALUE OF ADDITIONAL CAPITAL COST INCLUDING PARTS REPLACEMENTS OVER 20 YEARS
\$470	PRESENT VALUE OF ADDITIONAL MAINTENANCE COST (I.E., \$25 EVERY 5 YEARS PLUS \$25 EVERY YEAR, FOR 20 YEARS)
\$9,184	PRESENT VALUE OF TOTAL ADDITIONAL CAPITAL AND MAINTENANCE COST FOR THE SOLAR SYSTEM

---

[a] THE NOTATION \$/YRS. INDICATES THE COST AND FREQUENCY OF OCCURRENCE; E.G., \$25/5 YRS. INDICATES AN EXPENDITURE OF \$25 WHICH WILL HAVE TO BE DUPLICATED EVERY 5 YEARS.

[b] THIS ESTIMATE CONSISTS OF \$5.50 PER FT.^{2} FOR MATERIALS AND \$3.50 PER FT.^{2} FOR INSTALLATION, AND WAS SUGGESTED BY EXPERTS IN THE SOLAR ENERGY FIELD. IT FALLS IN THE LOWER HALF OF THE RANGE OF PRICES QUOTED AT A RECENT SOLAR INDUSTRY TRADE SHOW, WHERE PRICES GIVEN BY MAJOR PRODUCERS OF SOLAR COLLECTORS RANGED FROM \$3.50 TO ABOUT \$25.00 PER FT.^{2} INSTALLED. (SOLAR ENERGY INDUSTRIES ASSOCIATION INDUSTRY CONFERENCE AND TRADE SHOW, SHERATON PARK HOTEL, WASHINGTON, D.C., MAY 27 - 29, 1975.)

Table 4

PART C

Energy Costs

	ELECTRICITY COST TO DRIVE MOTORS & PUMPS	GALLONS #2 HEATING OIL REQUIRED	COSTS @ 40¢/GAL.
SOLAR	$20/YR.	436	$174/YR.
		(33,600,000 BTU ÷ 140,000 BTU/GAL. ÷ .55 BTU OUTPUT/BTU INPUT)	
CONVENTIONAL	$10/YR.	1,000	$400/YR.
		(84,000,000 ÷ 140,000 BTU/GAL. ÷ .60 BTU OUTPUT/BTU INPUT)	

$$\text{ANNUAL ENERGY COST SAVINGS}^{a} = \left[Q\left({}^{Pc/Bc}/Ec\right) + C\right] - \left[Q(1-S)\left({}^{Pa/Ba}/Ea\right) + D\right]$$

$$= \left[\$400 + \$10\right] - \left[\$174 + \$20\right]$$

$$= \$410 - \$194$$

$$= \$216$$

WHERE

Q = DESIRED QUANTITY OF BTU OUTPUT
Pc = PRICE PER UNIT OF FUEL FOR THE CONVENTIONAL SYSTEM,
Bc = BTU's SUPPLIED PER UNIT OF FUEL FOR THE CONVENTIONAL SYSTEM,
Ec = EFFICIENCY RATING OF THE CONVENTIONAL SYSTEM, I.E., BTU OUTPUT PER BTU INPUT,
C = OTHER ENERGY COSTS ASSOCIATED WITH THE CONVENTIONAL SYSTEM.
S = PERCENTAGE OF THE BTU DEMAND PROVIDED BY SOLAR,
Pa = PRICE PER UNIT OF FUEL FOR THE AUXILIARY SYSTEM,
Ba = BTU's SUPPLIED PER UNIT OF FUEL FOR THE AUXILIARY SYSTEM,
Ea = EFFICIENCY RATING OF THE AUXILIARY SYSTEM
D = OTHER ENERGY COSTS ASSOCIATED WITH THE SOLAR SYSTEM.

---

[a] ANNUAL SAVINGS WITHOUT ENERGY PRICE ESCALATION.

Table 4

PART D

Net Benefits Calculation, Assuming No Real Escalation in Fuel Prices

$(i = .02;\ e = 0)$

$$B_n = \left[F_c - F_s\right] - \left[C_s - C_c\right]$$

$$B_n = \left[(\$410 - \$194)\ \frac{(1 + .02)^{20} - 1}{.02\ (1 + .02)^{20}}\right] - \left[\$8{,}550 + \frac{\$200 + \$25}{(1 + .02)^{10}} + \frac{\$25}{(1 + .02)^{5}} + \frac{\$25}{(1 + .02)^{15}} + \$25 \cdot \left[\frac{(1 + .02)^{20} - 1}{.02\ (1 + .02)^{20}}\right]\right]$$

$$= \$3{,}532 - \$9{,}185$$

$$= -\$5{,}653$$

Table 4

PART E

Net Benefits Calculation, Assuming a 4% Real Increase in Fuel Prices

$$(i = .02; e = .04)$$

$$B_n = \left[F_c - F_s\right] - \left[C_s - C_c\right]$$

$$B_n = \sum_{j=1}^{20} \left[(\$410 - \$194)\left(\frac{1 + .04}{1 + .02}\right)^j\right] - \left[\$9,184\right]$$

$$= \$5,330 - \$9,184$$

$$= -\$3,855$$

auxiliary components, and fuel oil costs ($174) for the auxiliary system to provide 40% of required heating demands. Energy costs for the conventional system ($410) consist of electricity costs to power motors and pumps ($10), and fuel oil costs ($400) to provide 100% of required heating demand. The annual energy savings amounts to $216, i.e., $410 - 194.

Parts D and E of Table 4 show the calculation of net benefits of the solar system as compared with the conventional system, under two assumptions for energy cost escalation. Alternative assumptions are employed in the example in order to illustrate the sensitivity of the results to different rates of energy cost escalation (see Section 2.7 for a discusssion of sensitivity analysis). Part D calculates net benefits based on present estimated energy costs, with no escalation in real costs over time. Based on the assumptions of this hypothetical example, net benefits are shown to be a negative $5,653, and, accordingly, the life-cycle cost of the illustrative solar system exceeds that of the conventional system by this amount.

Part E of the exhibit calculates net benefits based on an increase in the real price of energy of 4%, compounded annually. The effect of this assumed escalation in energy costs is to raise substantially the energy savings of the solar system. However, the life-cycle cost of the solar system still exceeds that of the conventional system by $3,855.

Again, the reader is cautioned against placing undue emphasis on the numerical outcome of the case example. For a different set of assumptions, such as higher energy prices and/or lower cost of solar components, the outcome would be different. As research and development work proceeds, the costs of solar components may fall significantly from present levels.

2.7 Uncertainty in Cost Evaluations

As may be seen by the preceding example, evaluation results can be quite sensitive to both the data estimates and assumptions employed in the analysis. Factors affecting the outcome include (1) the discount rate used to convert future costs to an equivalent base; (2) the investment costs necessary to implement the HVAC systems; (3) the lives of the systems, their salvage values, and the length of the period over which the systems are compared; (4) the projections of future costs of maintenance and repair; (5) the rate of real price escalation in energy sources; (6) and the proportion of the total energy load provided by the different systems. The general direction of influence of these variables is summarized in Table 5.

The preceding discussions of evaluation methods have not dealt with the problem of uncertainty regarding the estimated values of costs and benefits to be used in the analysis. In fact however, alternative systems will involve varying degrees of uncertainty or risk[1] as to what will be their actual costs of acquisition and operation. Uncertainty regarding values for the solar system arises principally because it is a "new" technology. Uncertainty arises for the conventional system primarily because future availability and prices

---

[1]There are technical differences in the meaning of risk and uncertainty, but for the purpose here they can be treated as one, in that they both can cause results to vary from predictions. For a survey discussion of risk and uncertainty, see William Fellner, Probability and Profit: A Study of Economic Behavior Along Bayesian Lines (Homewood, Illinois: Richard D. Irwin, Inc., 1965, pp. 25-34).

Table 5

INFLUENCE OF ASSUMPTIONS ON ECONOMIC EFFICIENCY OF SOLAR ENERGY SYSTEMS

<u>DISCOUNT RATE</u>

HIGHER RATE → FEWER NET BENEFITS

<u>PERIOD OF ANALYSIS</u>

LONGER PERIOD → LARGER NET BENEFITS

<u>FUEL COST ESCALATION</u>

LARGER REAL COST RISE → LARGER NET BENEFITS

<u>SOLAR MATERIALS AND LABOR</u>

COST REDUCTIONS IN PRODUCTION, INSTALLATION, AND MAINTENANCE → LARGER NET BENEFITS

of fossil fuels are unknown. Thus, there are varying degrees of uncertainty attached to the estimated values of the cost items listed in Table 1.

One method of handling uncertainty in cost evaluations is to express costs and benefits as "expected values." This is done by multiplying the probability of an expected occurrence by the dollar value if the event does occur. For example, the probability of failure of the motors which circulate the collection medium through the collector plates might be multiplied by the cost of repairing the motors in order to find the expected cost of repairs.

Probability analysis requires determination of the probabilities attached to the various variables of each system--a difficult and uncertain effort in itself. Broad practical experience with solar systems or systematic research is needed to provide informed bases for estimating system probabilities. At present, the "best effort" will likely to be rely upon performance information provided by manufacturers, considered judgment of the analyst, and the use of break-even and sensitivity analysis to investigate the impact on costs of possible variations in the values of the determinant parameters.

Break-even analysis focuses on a single key variable which is regarded as a "risk factor." This form of analysis identifies the minimum (or maximum) value of the risk factor for which the alternatives would be equal, or which is required to achieve a targeted outcome. For example, one might solve for the break-even rate of escalation in fuel prices which would equate the life-cycle costs of a solar and a conventional system, given other costs. Alternatively, one might solve for the break-even collector plate price per sq. ft., or for the break-even number of years of use (i.e., the payback period). The analyst can then assess the likelihood of achieving less than or more than the break-even value for that factor.

Sensitivity analysis allows the analyst to determine the effect on the outcome of variation in one or more factors. A matrix can be developed to show the results of various combinations of assumed values for the determinant factors. Those factors which have a large impact on the outcome can thereby be identified and subjected to further study. Sensitivity analysis was used in the illustrative cost comparison of Section 2.6 to test the impact of alternative future changes in fuel prices on solar net benefits.

The appropriateness of assumptions to be used in evaluating alternative systems will vary depending upon the nature of use; for example, it might be reasonable to use different discount rates in evaluating HVAC systems used on commercial versus noncommercial buildings. However, it is important that assumptions be uniformly applied in making comparisons among systems. Table 6 shows an illustrative set of assumptions which might be used to evaluate life-cycle costs of alternative systems. A range is given for each variable to allow sensitivity assessments.

Table 6

ILLUSTRATIVE ASSUMPTIONS FOR EVALUATING SOLAR ENERGY SYSTEMS

Period of Analysis:	10 years 20 years 40 years
Real Discount Rate:	Low -- 2% (equivalent to a current market rate of about 10%) High -- 10% (equivalent to a current market rate of about 18%)
Cost Escalation:	Low -- No escalation[a]
	Medium -- No real escalation in non-energy costs Real escalation in heating oil & natural gas of 4% compounded annually[b] Real escalation in electricity of 1%, compounded annually
	High -- Real escalation in all fossil fuel energy sources of 10% compounded annually No real escalation in non-energy costs

---

[a] There is no consensus that energy prices will necessarily continue to rise at recent rates. In fact, a recent study suggests a decline in crude oil prices between 1974 and 1980. Paul Davidson, Lawrence H. Falk, and Haesung Lee, "Oil: Its Time Allocation and Project Independence," Brookings Papers on Economic Activity, Vol. 2, 1974 (Washington, D.C.: The Brookings Institution, p. 444).

[b] These escalation rates are approximately those estimated by William D. Nordhaus, "The Allocation of Energy Resources," Brookings Papers on Economic Activity, Vol. 3, 1973 (Washington, D.C.: The Brookings Institution, p. 55).

3. CONDITIONS FOR ECONOMIC OPTIMIZATION OF THE SOLAR HVAC SYSTEM AND THE BUILDING ENVELOPE

Section 2 presented methods of measuring the costs and comparing the efficiency of solar and conventional HVAC systems. These same methods of analysis can be applied to a number of tradeoffs in the design and operation of an HVAC system within a building. For example, at the system design level, life-cycle cost or benefit-cost analysis may be used to evaluate the efficiency of substituting among the various components and subsystems, such as increasing thermal storage capacity and decreasing collector area, or vice versa. In choosing among HVAC systems for a particular dwelling, evaluation may be made of the relative efficiency of various combinations of solar and auxiliary conventional systems. In designing or adapting a building for the use of a particular HVAC system, analysis techniques can be used to determine the efficiency of investing in energy conservation to reduce heat loads as compared with substituting a larger supply of heated or cooled air to the building. Analysis may also be made of tradeoffs among energy conservation techniques in order to determine the optimal combination of techniques to accomplish a given reduction in heating and cooling loads.[1] For example, different combinations of attic, wall, and floor insulation; storm windows and doors; insulating glasses; weather stripping; as well as building-site orientation and solar shading may be evaluated.

Since the alternatives will usually differ in their relative costs, the life-cycle costs of achieving a given heating and cooling objective may vary greatly depending upon the particular choices which are made. The evaluation techniques discussed in Section 2 can be used to evaluate the costs and benefits associated with incremental changes in the size or design of a system, and, therefore, are useful in optimizing the HVAC system within the building envelope.

The basic evaluation techniques having been treated, the focus of Section 3 is on setting forth and illustrating the conditions which are necessary for optimization, in the context of solar system/building design. These conditions and the basic cost-minimization approach which are outlined below could be applied to any of the tradeoff problems mentioned above. For purpose of illustrating the approach, however, emphasis is given in the discussion and examples to the problem of determining the least-cost combination of HVAC capacity and energy conservation measures which will meet a target level of comfort.

Two basic approaches to the problem of optimization are (1) minimization of total costs for a given output requirement, and (2) maximization of net benefits (i.e., finding the greatest difference between total benefits and total costs.) Where benefits are fixed, the two approaches give the same result and the choice between them is chiefly a matter of convenience given the nature of the particular problem. Since, for the problem at hand, all cash flows are negative and a level of comfort performance is given, the cost minimization approach will be discussed.[2]

---

[1]For a discussion and analysis of these tradeoffs, see, Steve Petersen, Retrofitting Existing Housing for Energy Conservation: An Economic Analysis.

[2]For more indepth discussions of economic optimization, see Henderson and Quandt, Microeconomic Theory: A Mathematical Approach (New York: McGraw-Hill Book Co., 1958), Chapter 3: and M. M. Bober, Intermediate Price and Income Theory (New York: W. W. Norton and Co., 1955), Chapter VI.

The problem of achieving a target comfort level by trading heating and cooling for energy conservation[1] is analogous to the classical economics problem of the "widget" manufacturer's need to determine the least-cost combination of labor and capital to produce a given quantity of widgets.

The designer of a solar dwelling may be viewed as the producer of a specified level of thermal comfort for a dwelling, where thermal comfort comprises temperature, humidity, or other related comfort attributes. The resources (inputs) to achieve the comfort objective consist of labor and material for the HVAC system, insulation, storm doors and windows, solar shading, and other energy conservation techniques, and also include various sources of energy.

The quantity of output, i.e., comfort level, can be expressed as $q = f(X_1, X_2, \ldots X_n)$, where $X_1, X_2, \ldots X_n$ are the quantities of the variable inputs. A production function could be specified that states the comfort level obtainable from every possible input combination. Many different combinations of inputs may be used to produce a given level of comfort. But the technical relationship between the inputs and the comfort level does not indicate the optimal combination of inputs to produce a given comfort level; the economically efficient input combination for the production of a particular comfort level depends upon the relative prices of the inputs.

A necessary condition for arriving at the minimal cost of producing a given comfort level is that the contribution of inputs towards achieving the desired comfort level be in the same proportion at the margin as their prices. This means that each input will be used up to that level at which its additional contribution to the objective per extra dollar spent is just equal to that for all other inputs. Assuming continuous and smooth functions, this necessary condition is expressible mathematically for two inputs as

$$\frac{\partial Q}{\partial X_1} \bigg/ \frac{\partial Q}{\partial X_2} = \frac{P_1}{P_2}\,, \qquad (17)$$

where

Q = units of output of comfort level,

X_1, X_2 = units of inputs 1 and 2, and

P_1, P_2 = cost per unit of inputs 1 and 2 in present value, life-cycle terms.

This expression could be expanded in a different form to accommodate as many inputs as are relevant.

This necessary condition is derived as follows: The decision maker attempts to minimize life-cycle costs (C_L), where

$$C_L = P_1X_1 + P_2X_2, \qquad (18)$$

---

[1]The higher the thermal resistance of the envelope, the less the change in the temperature of the interior air, and therefore, the smaller the need for additional heating or cooling of the space. Energy conservation actions to the building--by reducing conductive, convective, and radiation heat losses in winter and heat gains in summer--reduce the necessary load capacity and level of use of the HVAC system for a given building.

subject to the constraint that a specified level of comfort (Q_o) be met. Making the problem unconstrained by the Lagrange multiplier, we can minimize the expression

$$C_L = P_1X_1 + P_2X_2 + \lambda[Q_o - Q(X_1, X_2)]. \quad (19)$$

By setting the partial derivatives of C_L with respect to X_1 and X_2 equal to zero, we obtain

$$\frac{\partial C_L}{\partial X_1} = P_1 - \lambda \frac{\partial Q}{\partial X_1} = 0, \text{ and} \quad (20)$$

$$\frac{\partial C_L}{\partial X_2} = P_2 - \lambda \frac{\partial Q}{\partial X_2} = 0. \quad (21)$$

To derive the optimality condition shown in equation (17), we rewrite equations (20) and (21) as $P_1 = \lambda \frac{\partial Q}{\partial X_1}$ and $P_2 = \frac{\partial Q}{\partial X_2}$, divide the first equation by the second, and simplify, thus arriving at

$$\frac{\partial Q}{\partial X_1} \bigg/ \frac{\partial Q}{\partial X_2} = \frac{P_1}{P_2}. \quad (22)$$

This necessary condition[1] for the optimal combination of two inputs is illustrated graphically in Figure 1. Assume that the horizontal axis of Figure 1 measures the BTU load capacity of a given solar HVAC system (X_1); the vertical axis measures the quantity of a given package of energy conservation techniques (X_2). Curve Q_o indicates all the combinations of X_1 and X_2 which will yield a given level of comfort Q_o (assumed to be the target, i.e., minimum acceptable comfort level). For example, Q_o can be produced by combining the quantity $\overline{op}$ of X_1 and $\overline{oj}$ of X_2, or by combining $\overline{os}$ of X_1 and $\overline{og}$ of X_2. Moving from left to right down Q_o, we can determine on the horizontal axis the increase in the capacity of the solar HVAC system which would be necessary to offset a given reduction in the quantity of energy conservation techniques in order to maintain Q_o level of comfort. The curve shows the technical tradeoffs between the two inputs.

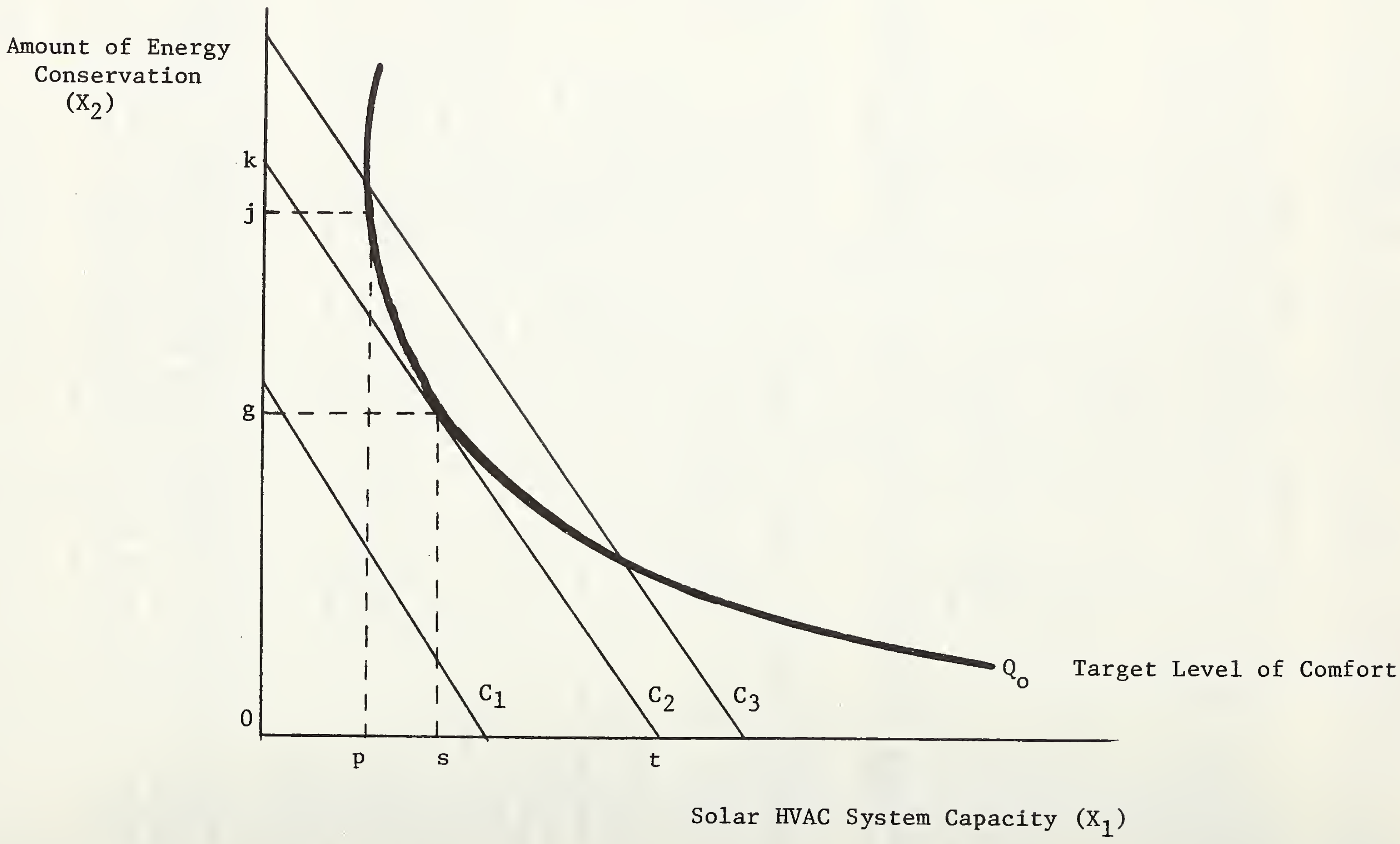
The lines C_1, C_2, and C_3 in Figure 1 illustrate three of a family of cost functions. Each function indicates a specific total cost, and shows the combinations of inputs 1 and 2 which may be purchased for that total cost. With the expenditure equal to C_2, for example, one could buy either $\overline{ot}$ of X_1 or $\overline{ok}$ of X_2 or any combination of X_1 and X_2 that lies on the cost line C_2. The slope of the cost line indicates the relative costs of the two inputs, i.e., P_1/P_2. Total costs rise as larger quantities of both inputs are purchased, that is, $C3 > C_2 > C_1$.

---

[1] The total benefits of each input must exceed its total costs in order to insure that the input is economical to add.

Figure 1

LEAST-COST COMBINATION



Given the relative costs of X_1 and X_2 indicated by the slopes of the cost curves, the lowest total cost at which Q_o comfort level can be produced is C_2, using $\overline{os}$ of X_1 in combination with $\overline{og}$ of X_2. For any cost less than C_2 (e.g., C_1) the desired comfort level, Q_o, could not be achieved. To achieve Q_o at any cost greater than C_2 (e.g., C_3), would be inefficient since Q_o can be achieved at the lower cost, C_2. Thus the least-cost combination of factors is determined by the point of tangency between the output curve, Q_o, and a cost curve. At the point of tangency,

$$\frac{\partial Q}{\partial X_1} \Big/ \frac{\partial Q}{\partial X_2} = \frac{P_1}{P_2},$$ and the basic optimality rule is met.

In practice, the optimal combination of HVAC/energy conservation inputs will vary with assumptions and given information. The particular optimal combination is dependent upon (1) climate factors, (2) comfort requirements, (3) functional characteristics of the building, (4) present and future energy costs, (5) costs of solar and conventional HVAC components, (6) costs of insulation and other energy conservation techniques, and (7) discount rates.

4. SUMMARY AND SUGGESTIONS FOR FURTHER RESEARCH

This paper addresses some economic issues important to the design, acquisition, and evaluation of solar heating and cooling systems. In Section 2, the paper explains and illustrates methods for evaluating and comparing the economic efficiency of solar and conventional heating and cooling systems for buildings. It identifies relevant costs, discusses data collection requirements, illustrates the discounting of costs, develops generalized life-cycle cost and benefit-cost models, sets forth techniques for developing models for unique problems, and discusses how the effective life-cycle costs to the owner of a solar-equipped building would be altered by current tax laws, insurance, and governmental incentives programs. Section 2 also discusses assumptions regarding the discount rate, the period of analysis, and the rate of price escalation in nonrenewable energy sources.

Section 3 of the paper sets forth the logic of and identifies an optimality rule for making cost-effective tradeoffs in the design of solar energy projects. For clarity and convenience, much of the discussion and illustration of optimality centers on the optimal tradeoffs between capacity of an HVAC system and investment in energy conservation in the building envelope.

While the paper treats a number of economic issues, it is not an exhaustive study of the economics of solar heating and coooling systems; nor does it meet the needs of all parties concerned with solar heating and cooling. In particular, the following tasks would appear to require further effort:

(1) Systematic investigation into the expected life-cycle costs of operating, maintaining, and repairing solar energy systems, and the projecting of future costs of solar system components, based on large scale production.

(2) Application of the evaluation techniques described in this paper to actual solar energy systems.

(3) A comprehensive analysis of the impact of laws, regulations, codes, zoning ordinances, and other practices on the costs of solar energy systems.

(4) Preparation of a consumer-oriented handbook to assist the homeowner and business person in making economic comparisons of alternative HVAC systems.

SELECTED BIBLIOGRAPHY

American Society for Testing and Materials. Metric Practice Guide. ASTM No. E380-72, 1972.

Bober, M. M., Intermediate Price and Income Theory. New York: W. W. Norton & Co., Inc., 1955.

Davidson, Paul, Falk, H. Lawrence, and Lee, Haesung. "Oil: Its Time Allocation and Project Independence." Brookings Papers on Economic Activity, Vol. 2. Washington, D.C.: The Brookings Institution, 1974.

DeGarmo, E. Paul. Engineering Economy, 4th Ed. New York: The MacMillan Co., 1967.

Fellner, William. Probability and Profit: A Study of Economic Behavior Along Bayesian Lines. Homewood, Illinois: Richard D. Irwin, Inc., 1965.

Grant, Eugene L. Principles of Engineering Economy. New York: The Ronald Press Company, 1950.

Grant, Eugene L., and Ireson, W. Grant. Principles of Engineering Economy, 5th Ed. New York: The Ronald Press Co., 1970.

Henderson, James M., and Quandt, Richard E. Microeconomic Theory; A Mathmatical Approach. New York: McGraw-Hill Book Co., 1958.

Nordhaus, William D. "The Allocation of Energy Resources." Brookings Papers on Economic Activity, Vol. 3. Washington, D.C.: The Brookings Institution, 1973.

Office of Management and Budget. Executive Office of the President. Circular No. A-94 (Revised), March 27, 1972.

Smith, Gerald W. Engineering Economy: Analysis of Capital Expenditures, 2nd Ed. Ames, Iowa: The Iowa State University Press, 1973.

U.S. Department of Commerce. National Bureau of Standards. Center for Building Technology. Retrofitting Existing Housing for Energy Conservation: An Economic Analysis, by Stephen R. Petersen, Washington, D.C., Building Science Series 64, December 1974.

NBS-114A (REV. 7-73)

U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET	1. PUBLICATION OR REPORT NO. NBSIR 75-712	2. Gov't Accession No.	3. Recipient's Accession No.
4. TITLE AND SUBTITLE Solar Heating and Cooling of Buildings: Methods of Economic Evaluation			5. Publication Date May 1975
			6. Performing Organization Code 463.02
7. AUTHOR(S) Rosalie T. Ruegg			8. Performing Organ. Report No. NBSIR 75-712
9. PERFORMING ORGANIZATION NAME AND ADDRESS NATIONAL BUREAU OF STANDARDS DEPARTMENT OF COMMERCE WASHINGTON, D.C. 20234			10. Project/Task/Work Unit No.
			11. Contract/Grant No.
12. Sponsoring Organization Name and Complete Address (Street, City, State, ZIP) National Bureau of Standards Department of Commerce Washington, D.C. 20234			13. Type of Report & Period Covered Final
			14. Sponsoring Agency Code

15. SUPPLEMENTARY NOTES

16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

This report addresses economic issues important to the design, and evaluation of solar heating and cooling systems in buildings. It explains and illustrates with simple, but realistic examples the use of life-cycle cost analysis and benefit-cost analysis to evaluate and compare the economic efficiency of solar and conventional energy systems. It also explains the conditions for making cost-effective tradeoffs in solar system/building design. By presenting the basic methods and assessing the appropriateness of alternative assumptions, the paper provides a resource document for researchers and analysts.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Solar energy; solar heating and cooling; HVAC systems; life-cycle cost analysis; economic optimization

18. AVAILABILITY [X] Unlimited	19. SECURITY CLASS (THIS REPORT) UNCLASSIFIED	21. NO. OF PAGES 47
[ ] For Official Distribution. Do Not Release to NTIS		
[ ] Order From Sup. of Doc., U.S. Government Printing Office Washington, D.C. 20402, SD Cat. No. C13	20. SECURITY CLASS (THIS PAGE) UNCLASSIFIED	22. Price $3.75
[X] Order From National Technical Information Service (NTIS) Springfield, Virginia 22151		

USCOMM-DC 29042-P74

AMERICAN REVOLUTION BICENTENNIAL
1776-1976